Student Number___________________

ASCHAM SCHOOL

2021

YEAR 12

TRIAL

EXAMINATION

Mathematics
Extension 2

General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours.
- Write using black non-erasable pen.
- NESA-approved calculators may be used.
- A NESA Reference Sheet is provided.
- All necessary working should be shown in every question.

Total marks – 100

- Attempt Sections A and B.
- Section A is worth 10 marks.
- Recommended time on Section A: 15 minutes
- Answer Section A on the multiple choice answer sheet.
- Detach the multiple choice answer sheet from the back of the examination paper.
- Section B contains 6 questions worth 15 marks each.
- Recommended time on Section B: 2 hours 45 minutes
- Answer each question in a new booklet.
- Label all sections clearly with your name/number and teacher.

SECTION A – 10 MULTIPLE CHOICE QUESTIONS 10 MARKS

ANSWER ON THE ANSWER SHEET

1 Which of the following is closest to the angle between the vector $\underset{\sim}{u} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$ and the y-axis?

A 24°
B 36°
C 61°
D 109°

2 Which of the following is equivalent to the value of $\dfrac{2i}{e^{\frac{5\pi}{6}i}}$?

A $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

B $2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$

C $-2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

D $2\left(\cos\left(-\frac{2\pi}{3}\right) - i\sin\left(-\frac{2\pi}{3}\right)\right)$

3 Which of the following is true?

A $\forall x, y \in \mathbb{N}, \exists z \in \mathbb{Q} : z\sqrt{y} = \dfrac{x}{\sqrt{y}}$

B $\forall x, y \in \mathbb{R}, \exists z \in \mathbb{N} : z\sqrt{y} = \dfrac{x}{\sqrt{y}}$

C $\forall x, y \in \mathbb{Q}, \exists z \in \mathbb{Z} : z\sqrt{y} = \dfrac{x}{\sqrt{y}}$

D $\forall x, y \in \mathbb{Z}, \exists z \in \mathbb{R} : z\sqrt{y} = \dfrac{x}{\sqrt{y}}$

4 Which of the following is a root of $z^5+1=0$?

A $\cos\left(\dfrac{\pi}{5}\right)+i\sin\left(\dfrac{\pi}{5}\right)$

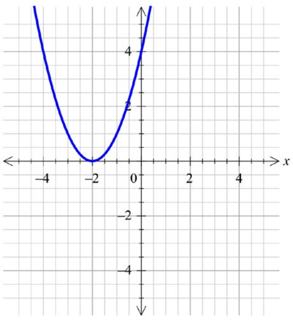
B $\cos\left(\dfrac{2\pi}{5}\right)+i\sin\left(\dfrac{2\pi}{5}\right)$

C $\cos\left(-\dfrac{2\pi}{5}\right)+i\sin\left(-\dfrac{2\pi}{5}\right)$

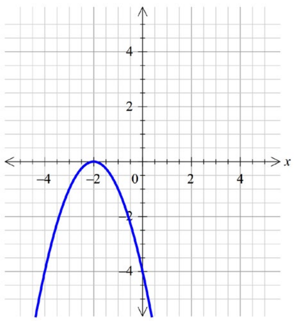
D $\cos\left(-\dfrac{4\pi}{5}\right)+i\sin\left(-\dfrac{4\pi}{5}\right)$

5 What is a possible graph of v^2 versus x for a particle moving in simple harmonic motion?

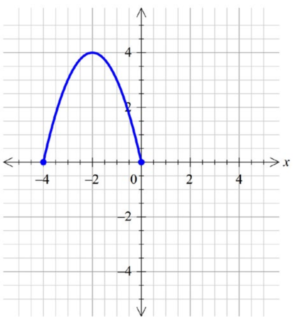
A



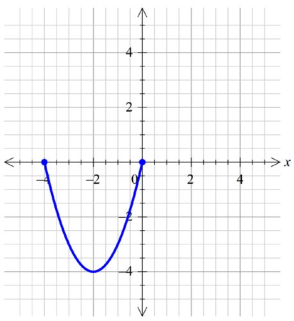
B



C



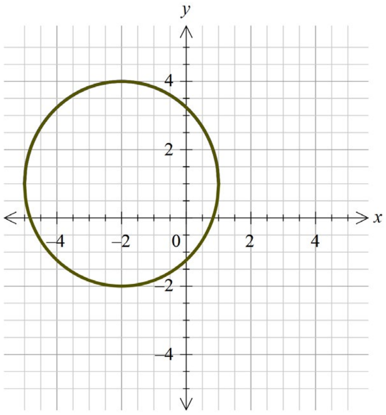
D



6 Which of the following is a counter-example to the following statement?
If you are over 60 then you must get the AstraZeneca vaccine.

A If you are under 60 then you must not get the AstraZeneca vaccine.
B Jo is over 60 and got the Pfizer vaccine instead.
C Polly is under 60 and got the AstraZeneca vaccine.
D If you get the AstraZeneca vaccine you must be over 60.

7 The graph below is on the Argand diagram.



What is the equation of the graph?

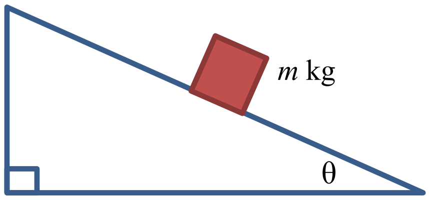
A $|z-2-i|=3$

B $|z-2+i|=9$

C $|z+2-i|=3$

D $|z+2+i|=9$

8 A mass m kg is resting on a slope inclined at θ to the horizontal. Forces acting are the normal force N, a frictional force F and the gravitational force mg.



Which of the following is the correct resolution of forces in the horizontal direction?

A $F\cos\theta - N\sin\theta = 0$
B $F\sin\theta - N\sin\theta = 0$
C $F\sin\theta - N\cos\theta = 0$
D $F\cos\theta - N\cos\theta = 0$

9 A particle P is moving in simple harmonic motion. Its maximum speed is 6π m/s and its displacement then is 4 m. After 10 seconds it has reached its maximum speed again. What is a possible equation for the displacement of P?

A $x = 60\sin\left(\dfrac{\pi t}{10}\right) + 4$

B $x = 60\sin\left(\dfrac{\pi t}{20}\right) + 4$

C $x = 60\sin(10t) + 4$

D $x = 60\sin(20t) + 4$

10 Consider the complex numbers $z_1 = 1 - i$ and $z_2 = 1 + i\sqrt{3}$. What is the smallest positive value of n such that $\dfrac{(z_2)^n}{(z_1)^n}$ is purely imaginary?

A 2
B 4
C 6
D 8

SECTION 2 – 6 QUESTIONS EACH WORTH 15 MARKS

Question 11 – Begin a new writing booklet

a Find $\int \frac{x+1dx}{x^2+4}$. **3**

b Find $\int_1^2 \frac{1}{x(x+5)}dx$. **4**

c Find $\int \tan^6 x \; dx$ **4**

.

d Calculate the modulus and argument of the product of the roots of the equation **4**
$(5+3i)z^2-(1-4i)z+(8-2i)=0$

.

Question 12 – Begin a new writing booklet

a Consider the statement for $p \in \mathbb{N}$:

If p has an odd number of distinct factors then p is a square number.

i Write the converse. **1**

ii Write the contrapositive. **2**

iii Write the negation. **2**

iv Determine whether or not the statement is an equivalence. Give reasons. Is the statement true? **2**

b Let $z = x+iy$ where $x, y \in \mathbb{R}$. Sketch on an Argand diagram the points satisfying $z^2-\overline{z}^{\,2}=8i$. **2**

c Find all complex numbers $z = x+iy$ where $x, y \in \mathbb{R}$ such that $|z|^2+5\overline{z}+10i=0$ **3**

.

d Let $z_1, z_2 \in \mathbb{C}$ such that $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$. Prove that $|z_1||z_2| = |z_1 z_2|$. **3**

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Question 13 – Begin a new writing booklet

a The 4 complex numbers z_1, z_2, z_3, z_4 are represented by the points Z_1, Z_2, Z_3, Z_4 on an Argand diagram. If $z_1 - z_2 + z_3 - z_4 = 0$ and $z_1 - iz_2 - z_3 + iz_4 = 0$, what type of quadrilateral could $Z_1Z_2Z_3Z_4$ be? **3**

b Consider the line l_1 with equation $\dfrac{x-2}{3} = \dfrac{y+4}{2} = \dfrac{z-1}{-4}$.

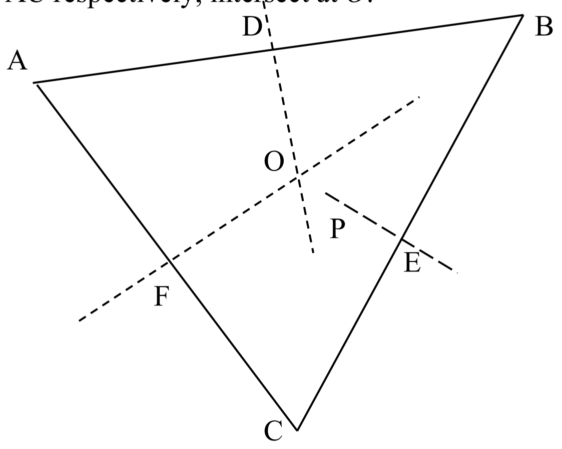
i Find the vector equation of any line l_2 that passes through the point $P(1,2,3)$, which is perpendicular to l_1. Is this solution unique? **3**

ii If the lines l_1 and l_2 intersect, find the point of intersection. **3**

Question 13 continues on the next page.

Question 13 continued

c Consider the triangle *ABC* shown. The perpendicular bisectors, *DO* and *FO,* of *AB* and A*C* respectively, intersect at *O*.



Copy the diagram.

Let $\overrightarrow{OA} = \underset{\sim}{a}, \overrightarrow{OB} = \underset{\sim}{b}, \overrightarrow{OC} = \underset{\sim}{c}, \overrightarrow{OD} = \underset{\sim}{d}$, *etc.*

Use vectors to:

i prove that $\underset{\sim}{d} = \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$ **1**

ii prove that *A,B* and *C* all lie on a circle with centre *O*, ie $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right| = \left|\overrightarrow{OC}\right|$ **2**

iii prove that the perpendicular bisector of BC, that is, EP passes through *O*. **3**

Question 14 – Begin a new writing booklet

a **i** If *a, b* are real, show that $\dfrac{a^2+b^2}{2} \geq ab$. **1**

ii Hence, or otherwise, show that if *a, b, c, d* are real and $a, b, c, d \geq 0$ then **2**

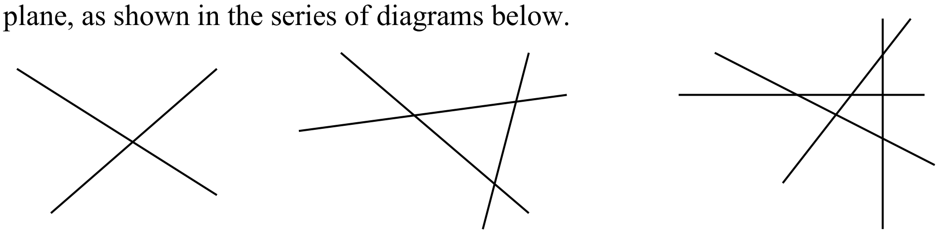
$\sqrt{\left(a^2+c^2\right)\left(b^2+d^2\right)} \geq ab+cd$.

b Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ using the substitution $u = a - x$. **1**

Hence evaluate $\int_0^2 x(2-x)^{10}\,dx$. **3**

c Prove by the contrapositive that if *m* and *n* are positive integers such that $m+n$ and $m-n$ have no common factors, then *m* and *n* have no common factors. **3**

d Consider the maximum number *N* of possible points of intersection of *n* lines on a plane, as shown in the series of diagrams below.



$n = 2$ $\quad$ $n = 3$ $\quad$ $n = 4$

i Copy and complete the table: **2**

n	2	3	4	5	6
N	1	3			

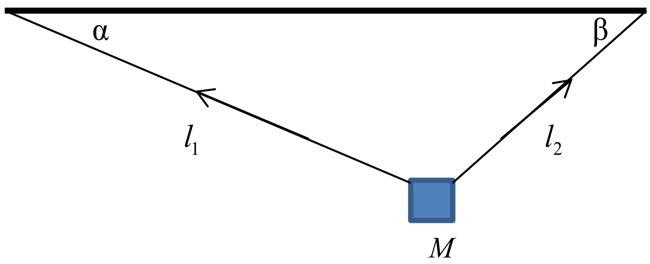
ii Prove by induction that the maximum number *N* of points of intersection of *n* lines is **3**

$$N = \frac{n(n-1)}{2} \text{ for } n \geq 2.$$

Question 15 – Begin a new writing booklet

a Find the four 4th roots of $-\dfrac{1}{2}+\dfrac{i\sqrt{3}}{2}$ and show them on an Argand diagram. **4**

b A body of mass M kg is being suspended by two strings of differing lengths l_1 and l_2 attached to the mass and then to either end of a horizontal rod as shown in the diagram. Acceleration due to gravity is g m/s^2. **4**



The tensions in l_1 and l_2 are T_1 and T_2 (Newtons) respectively.
Show that the tension T_1 in the string l_1 is given by
$T_1 = \dfrac{Mg\cos\beta}{\sin(\alpha+\beta)}$.

c Let $I_n = \displaystyle\int \frac{dx}{\left(x^2+1\right)^n}$.

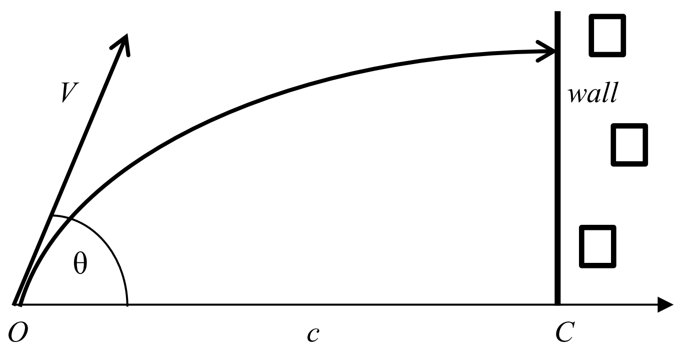
i Show that $\displaystyle\int \frac{dx}{\left(x^2+1\right)^n} = \int \frac{dx}{\left(x^2+1\right)^{n-1}} - \int \frac{x^2\,dx}{\left(x^2+1\right)^n}$. **1**

ii Use the identity above to find constants A and B such that $I_n = \dfrac{Ax}{\left(x^2+1\right)^{n-1}} + BI_{n-1}$. **4**

iii Hence, or otherwise find $\displaystyle\lim_{k\to\infty}\int_0^k \frac{dx}{\left(x^2+1\right)^2}$. **2**

Question 16 – Begin a new writing booklet

a A hose is at O and aimed to hit a vertical wall which is c metres along the horizontal ground from O. The hose water comes out at a speed V m/s and is aimed at an angle θ from the horizontal where $0° < \theta < 90°$. Take g m/s^2 as acceleration due to gravity and ignore air resistance. **4**



Show that the hose water will reach the wall if and only if $V > \sqrt{gc}$.

b **i** Show that $1 + r + r^2 + r^3 + ... + r^{n-1} = \dfrac{1 - r^n}{1 - r}$ for $r \neq 1$. **1**

ii As $n \to \infty$, find an expression for $1 + r + r^2 + r^3 + ... + r^{n-1}$ for $0 < r < 1$. **1**

iii Prove that for $0 < x < 1$, $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \dfrac{x^4}{4} + ... + \dfrac{x^n}{n} + ... = -\ln(1 - x)$. **2**

Question 16 continues on the next page.

Question 16 continued

c Two particles P_1 and P_2 are moving with velocities v_1 and v_2 respectively in Simple Harmonic Motion along the same axis according to the equations

$$x_1 = A + \sin nt$$
$$\ddot{x}_2 = -n^2 x_2$$
where $n, A \in \mathbb{R}$ and $n, A > 0$, $t \geq 0$.

Initially, $x_1 = A$, $x_2 = 1$ and $v_2 = 0$.

Assume the value of A is such that the particles never meet.

i Show that $\dot{x}_1^{\,2} = n^2\left(1 - \left(x_1 - A\right)^2\right)$. **2**

ii Show that $\dot{x}_2^{\,2} = n^2\left(1 - x_2^{\,2}\right)$. **2**

iii Find the minimum distance between the two particles. **3**

The end! ☺

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Student Number ....

ASCHAM SCHOOL

YEAR 12 Trial Mathematics Extension 2 Exam

MULTIPLE-CHOICE ANSWER SHEET

1.	A ⬭	B ⬭	C ⬭	D ⬭
2.	A ⬭	B ⬭	C ⬭	D ⬭
3.	A ⬭	B ⬭	C ⬭	D ⬭
4.	A ⬭	B ⬭	C ⬭	D ⬭
5.	A ⬭	B ⬭	C ⬭	D ⬭
6.	A ⬭	B ⬭	C ⬭	D ⬭
7.	A ⬭	B ⬭	C ⬭	D ⬭
8.	A ⬭	B ⬭	C ⬭	D ⬭
9.	A ⬭	B ⬭	C ⬭	D ⬭
10.	A ⬭	B ⬭	C ⬭	D ⬭

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Solutions to

Title: Ascham 2021 Math Ext 2 TRIAL

MULTIPLE CHOICE

1. y-axis is $(0,1,0)$ say.

$\underset{\sim}{u}\cdot\underset{\sim}{y} = |\underset{\sim}{u}||\underset{\sim}{y}|\cos\theta$

$$\therefore \cos\theta = \frac{3(0)+5(1)-2(0)}{\sqrt{3^2+5^2+(-2)^2}\sqrt{0^2+1^2+0^2}}$$

$$= \frac{5}{\sqrt{38}}$$

(B)

$\theta \approx 36^\circ$

2. $$\frac{2i}{e^{\frac{5\pi}{6}i}} = \frac{2e^{\frac{\pi}{2}i}}{e^{\frac{5\pi}{6}i}}$$

$$= 2e^{i\left(\frac{\pi}{2}-\frac{5\pi}{6}\right)}$$

$$= 2e^{i\left(-\frac{\pi}{3}\right)}$$

$$= 2\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right)$$

$$= 2\left(\cos\frac{\pi}{3} - i\sin\left(\frac{\pi}{3}\right)\right)$$

$$= 2\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$$

$$= 2\left(-\cos\frac{2\pi}{3} - i\sin\left(\frac{2\pi}{3}\right)\right)$$

$= $ (C)

3. $z\sqrt{y} = \dfrac{x}{\sqrt{y}}$ or $zy = x$ if $y > 0$

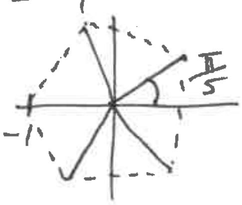
(A)

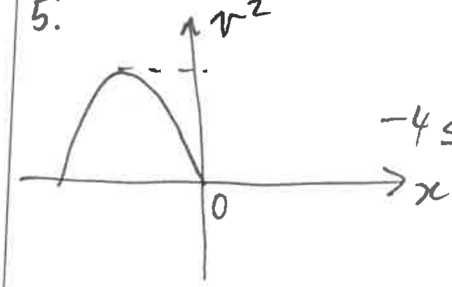
or $z = \dfrac{x}{y}$

$\forall x,y \in \mathbb{N}, \exists z \in \mathbb{Q} : z\sqrt{y} = \frac{x}{\sqrt{y}}$.

4. $z^5 + 1 = 0$. $z^5 = -1$

(A)



5. 

$v^2 \geq 0$

$-4 \leq x \leq 0$

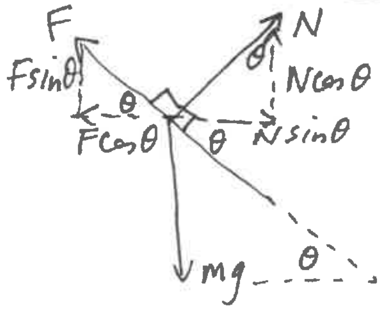
(C)

6. (B) Jo is over 60 and did not get the AstraZ vaccine.

7. Centre $(-2,1)$ $-2+i$, $r=3$

$|z-(-2+i)| = 3$

$|z+2-i| = 3$ (C)

8. 

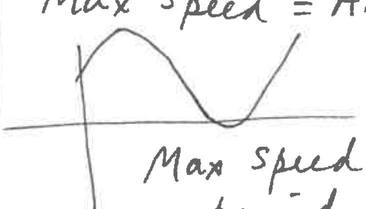
(A)

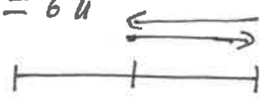
Horizontal: $F\cos\theta - N\sin\theta = 0$

9. $x = A\sin nt + k$

$\dot{x} = An\cos nt$ $\quad -1 \leq \cos nt \leq 1$

Max speed $= An = 6\pi$





Max speed difference $= \frac{1}{2}$ period. $= 10$ sec

Period $= \dfrac{2\pi}{n}$

$20 = \dfrac{2\pi}{n}$ $\therefore$ $n = \dfrac{2\pi}{20} = \dfrac{\pi}{10}$.

$\therefore A\left(\frac{\pi}{10}\right) = 6\pi$ $\therefore A = 60$.

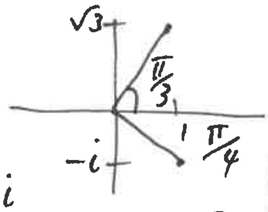
$\therefore x = 60\sin\left(\frac{\pi}{10}t\right) + 4$ (A)

PTO

MULT. CHOICE cont'd

10. $z_1 = 1 - i \quad , \quad z_2 = 1 + i\sqrt{3}.$

$$\frac{(z_2)^n}{(z_1)^n} = ki$$



$$\left(\frac{1+i\sqrt{3}}{1-i}\right)^n = ki$$

$k \in \mathbb{R}$

$$\left(\frac{2 \operatorname{cis}\frac{\pi}{3}}{\sqrt{2}\operatorname{cis}\frac{\pi}{4}}\right)^n = \left(\sqrt{2}\operatorname{cis}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)\right)^n$$

$$= (\sqrt{2})^n \left(\operatorname{cis}\left(\frac{\pi}{12}\right)\right)^n$$

$$ki = (\sqrt{2})^n \left(\operatorname{cis}\frac{n\pi}{12}\right)$$

$\operatorname{cis}\frac{\pi}{2} = \operatorname{cis}\frac{6\pi}{12}$ is imaginary

$\therefore n = 6.$ C

Solutions to
Title: Ascham 2021 Math Ext 2 TRIAL

Q11

a) $\int \frac{x+1\,dx}{x^2+4} = \int \frac{x}{x^2+4} + \frac{1}{x^2+4}\,dx$

3

$= \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\tan^{-1}\frac{x}{2} + c$

b) $\int_1^2 \frac{1}{x(x+5)}\,dx = \int_1^2 \frac{A}{x} + \frac{B}{x+5}\,dx$

So $1 \equiv A(x+5) + Bx$

$x=-5 \quad 1 = Bx-5 \Rightarrow B = -\frac{1}{5}$

$x=0 \quad 1 = 5A \Rightarrow A = \frac{1}{5}$

$\therefore \int_1^2 \frac{1}{x(x+5)}\,dx = \int_1^2 \frac{1}{5}\frac{1}{x} - \frac{1}{5}\frac{1}{x+5}\,dx$

$= \frac{1}{5}\Big[\ln|x| - \ln|x+5|\Big]_1^2$

4

$= \frac{1}{5}\Big[\ln|2| - \ln|7| - (\ln|1| - \ln|6|)\Big]$

$= \frac{1}{5}\left(\ln|2| - \ln|7| + \ln|6|\right)$

$= \frac{1}{5}\ln\left|\frac{12}{7}\right|$

c) $\int \tan^6 x\,dx = \int \tan^4 x . \tan^2 x\,dx$

$= \int \tan^4 x\,(\sec^2 x - 1)\,dx$ 4

$= \int \tan^4 x \sec^2 x - \int \tan^4 x\,dx$

$= \frac{\tan^5 x}{5} - \int \tan^2 x . (\sec^2 x - 1)\,dx$

$= \frac{\tan^5 x}{5} - \int \tan^2 x \sec^2 x - \tan^2 x\,dx$

$= \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} - \int \sec^2 x - 1\,dx$

$= \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x - x + c$

d) $(5+3i)z^2 - (1-4i)z + (8-2i) = 0$

Product of roots $= \frac{c}{a}$

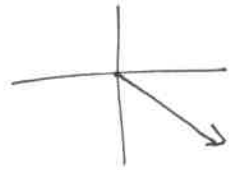
$= \frac{8-2i}{5+3i} \times \frac{5-3i}{5-3i}$

$= \frac{40 - 6 - 10i - 24i}{25+9}$

4

$= \frac{34 - 34i}{34}$

$= 1 - i$



$|1-i| = \sqrt{1+1}$

$= \sqrt{2}$

$\arg(1-i) = -\frac{\pi}{4}$.

Q12

a) i) Converse of $P \Rightarrow Q$ is $Q \Rightarrow P$.
If p is a square number then p has an odd number of distinct factors.

ii) Contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$. 1

If p is not a square number then p does not have an odd number of distinct factors. 2

iii) The negation: 2
There exists a number p with an odd number of distinct factors such that p is not a square number.

iv) It is an equivalence because both $P \Rightarrow Q$ AND $\neg Q \Rightarrow \neg P$ are true. The statement is true. 2
eg. $36 = 6^2$. Factors come in pairs except for 6:
1, 36, 18, 2, 12, 3, 9, 4, 6.

b) $z^2 - \bar{z}^2 = 8i$. 2

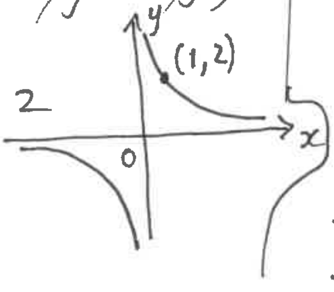
$(x+iy)^2 - (x-iy)^2 = 8i$

$(\cancel{x}+iy-\cancel{x}+iy)(x+\cancel{iy}+x-\cancel{iy}) = 8i$

$2iy \times 2x = 8i$

$\cancel{4i}xy = \cancel{8i}\ 2$

$xy = 2$.



c) $|z|^2 + 5\bar{z} + 10i = 0$

$|x+iy|^2 + 5(x-iy) = -10i$

$x^2+y^2+5x-5iy+10i=0$

Equate reals and imaginaries:

$x^2+y^2+5x=0$, $\quad -5\cancel{i}y+10\cancel{i}=0$

$5y = 10$

$y = 2$ 3

$x^2+2^2+5x=0$

$x^2+5x+4=0$

$(x+4)(x+1)=0$

$x=-4$ or $x=-1$, $y=2$.

$\therefore z = -4+2i$, or $-1+2i$

d) RTP: $|z_1||z_2| = |z_1 z_2|$

Proof: $LHS = |z_1||z_2|$ 3

$= |x_1+iy_1||x_2+iy_2|$

$= \sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}$

$= \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$

$RHS = |z_1 z_2|$

$= |(x_1+iy_1)(x_2+iy_2)|$

$= |x_1x_2 + x_1iy_2 + x_2iy_1 + -y_1y_2|$

$= |(x_1x_2 - y_1y_2) + i(x_1y_2+x_2y_1)|$

$= \sqrt{(x_1x_2-y_1y_2)^2 + (x_1y_2+x_2y_1)^2}$

$= \sqrt{x_1^2x_2^2 \cancel{-2x_1x_2y_1y_2} + y_1^2y_2^2 \ldots}$
$\overline{\ldots + x_1^2y_2^2 \cancel{+2x_1x_2y_1y_2} + x_2^2y_1^2}$

$= \sqrt{x_1^2x_2^2 + x_1^2y_2^2 + y_1^2y_2^2 + y_1^2x_2^2}$

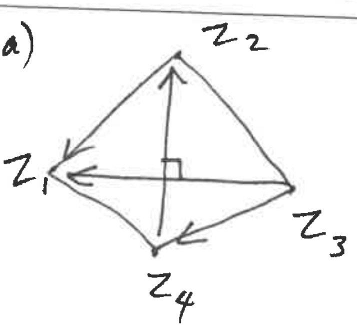
$= \sqrt{x_1^2(x_2^2+y_2^2) + y_1^2(y_2^2+x_2^2)}$

$= \sqrt{(x_2^2+y_2^2)(x_1^2+y_1^2)}$

$= LHS$

$\therefore |z_1||z_2| = |z_1 z_2|$

13 a)



3

$z_1 - z_2 + z_3 - z_4 = 0$

$\therefore z_1 - z_2 = z_4 - z_3$

$\therefore z_1 - z_2$ is parallel to $z_4 - z_3$

and $|z_1 - z_2| = |z_4 - z_3|$

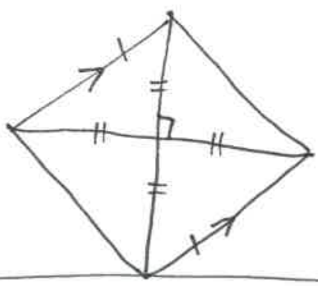
$\therefore z_1 z_2 z_3 z_4$ has one pair of opposite sides equal and parallel $\therefore$ could be a parm.

Also, $z_1 - iz_2 - z_3 + iz_4 = 0$

$\therefore z_1 - z_3 = i(z_2 - z_4)$

$\therefore z_1 - z_3$ is same as $z_2 - z_4$ only perpendicular $\therefore$

$z_1 z_2 z_3 z_4$ has equal diagonals that are perpendicular as well as a pair of opposite sides equal and parallel



It must be a square.

b) $l_1: \dfrac{x-2}{3} = \dfrac{y+4}{2} = \dfrac{z-1}{-4} = \lambda$

i) $l_1:$ $x = 3\lambda + 2$
$y = 2\lambda - 4$
$z = -4\lambda + 1$

3

or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$

Let direction be $\perp$ to l_1 $\therefore$

l_2 direction is $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ where

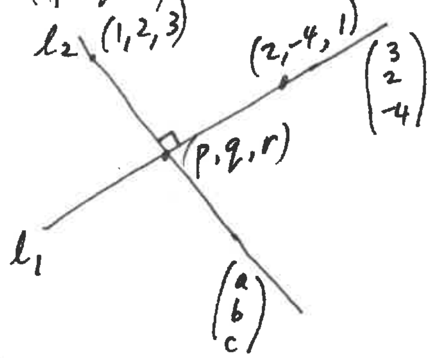
$\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$ ie $3a + 2b - 4c = 0$

Say $a = 2, b = 3, c = 3$

then l_2 is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$.

Not a unique solution.

ii) l_1 and l_2 intersect at say (p, q, r) and $l_1 \perp l_2$ so



3

$\exists\, a, b, c$ such that $a = p - 1$
$b = q - 2$
$c = r - 3$

and $3a + 2b - 4c = 0$

and $\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$

$\therefore$ $p = 2 + 3\lambda$
$q = -4 + 2\lambda$
$r = 1 - 4\lambda$

so $a = 2 + 3\lambda - 1$
$= 1 + 3\lambda$
$b = -4 + 2\lambda - 2$
$= -6 + 2\lambda$
$c = 1 - 4\lambda - 3$
$= -2 - 4\lambda$

$\therefore 3a + 2b - 4c = 0$

$3(1 + 3\lambda) + 2(-6 + 2\lambda) - 4(-2 - 4\lambda) = 0$

$3 + 9\lambda - 12 + 4\lambda + 8 + 16\lambda = 0$

$-1 + 29\lambda = 0$

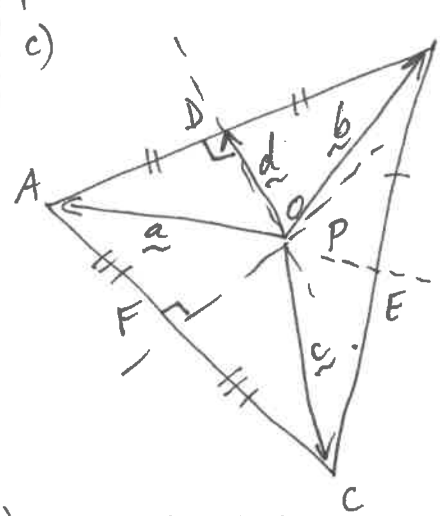
$\therefore \lambda = \dfrac{1}{29}$

$\therefore$ Point is $\left(2 + \dfrac{3}{29}, -4 + \dfrac{2}{29}, 1 - \dfrac{4}{29}\right)$

$\left(2\dfrac{3}{29}, -3\dfrac{27}{29}, \dfrac{25}{29}\right)$.

Q13 Cont'd

c)



i) RTP: $\underset{\sim}{d} = \frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$

Proof: Given $\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB}$

$\therefore \underset{\sim}{d} - \underset{\sim}{a} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

$\underset{\sim}{d} = \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a} + \underset{\sim}{a}$ (1)

$= \frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$ QED!!!

ii) RTP: $|\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OC}|$

Proof: Given $\overrightarrow{OD} \perp \overrightarrow{AB}$

then $\overrightarrow{OD} \cdot \overrightarrow{AB} = 0$

$(\underset{\sim}{d} - 0)\cdot(\underset{\sim}{b} - \underset{\sim}{a}) = 0$

$\therefore \frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{b}-\underset{\sim}{a}) = 0$

$\frac{1}{2}(\underset{\sim}{b}\cdot\underset{\sim}{b} - \underset{\sim}{a}\cdot\underset{\sim}{a}) = 0$

$\therefore |\underset{\sim}{b}|^2 - |\underset{\sim}{a}|^2 = 0$

$\therefore |\underset{\sim}{b}|^2 = |\underset{\sim}{a}|^2$ $(|a|,|b|>0)$

$\therefore |\underset{\sim}{b}| = |\underset{\sim}{a}|$

$\therefore |\overrightarrow{OB}| = |\overrightarrow{OA}|$.

Similarly we can see that

$\underset{\sim}{f} = \frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{c})$

Given $\overrightarrow{OF} \perp \overrightarrow{AC}$

then $\overrightarrow{OF} \cdot \overrightarrow{AC} = 0$

$\therefore f\cdot(\underset{\sim}{c} - \underset{\sim}{a}) = 0$

$\therefore \frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{c})\cdot(\underset{\sim}{c}-\underset{\sim}{a}) = 0$

$\therefore \underset{\sim}{c}\cdot\underset{\sim}{c} - \underset{\sim}{a}\cdot\underset{\sim}{a} = 0$ (2)

$\therefore |c|^2 = |a|^2$

$\therefore |\overrightarrow{OC}| = |\overrightarrow{OA}| = |\overrightarrow{OB}|$

QED!

iii) RTP: $\overrightarrow{PE} = \lambda\overrightarrow{EO}$.

Proof: We know that $\underset{\sim}{e} = \frac{1}{2}(\underset{\sim}{c}+\underset{\sim}{b})$.

Also $|\overrightarrow{OC}| = |\overrightarrow{OB}|$.

If $\overrightarrow{EP}$ is the perp. bisector then

$\overrightarrow{EP}\cdot\overrightarrow{BC} = 0$

$\therefore (\underset{\sim}{p} - \underset{\sim}{e})\cdot(\underset{\sim}{c} - \underset{\sim}{b}) = 0$

$\therefore (\underset{\sim}{p} - \frac{1}{2}(\underset{\sim}{c}+\underset{\sim}{b}))\cdot(\underset{\sim}{c}-\underset{\sim}{b}) = 0$

$\therefore \underset{\sim}{p}\cdot(\underset{\sim}{c}-\underset{\sim}{b}) - \frac{1}{2}(c.c - b.b) = 0$

$\therefore p\cdot(\underset{\sim}{c}-\underset{\sim}{b}) - \frac{1}{2}(|c|^2 - |b|^2) = 0$

but $|c|^2 = |b|^2$

$\therefore p\cdot(\underset{\sim}{c}-\underset{\sim}{b}) - \frac{1}{2}(0) = 0$ (3)

$\therefore p\cdot(\underset{\sim}{c}-\underset{\sim}{b}) = 0$

$\therefore (\underset{\sim}{p} - 0)\cdot(\underset{\sim}{c}-\underset{\sim}{b}) = 0$

$\therefore \overrightarrow{OP} \perp \overrightarrow{CB}$

but $\overrightarrow{EP} \perp \overrightarrow{CB}$ as well

$\therefore$ P must lie on $\overrightarrow{OE}$

or $\overrightarrow{PE} = \lambda\overrightarrow{EO}$

$\therefore$ EP passes through O.

Solutions to
Title: Ascham 2021 Math Ext 2 TRIAL

Q14

a i) RTP: $\frac{a^2+b^2}{2} \geq ab$

Proof: Consider the difference:

$\frac{a^2+b^2}{2} - ab = \frac{a^2+b^2-2ab}{2}$

$= \frac{(a-b)^2}{2}$ (1)

≥ 0 since $(a-b) \geq 0$

$\therefore \frac{a^2+b^2}{2} \geq ab.$

ii) RTP: $\sqrt{(a^2+c^2)(b^2+d^2)} \geq ab+cd.$

Proof: Using the fact that if $\sqrt{P} \geq Q$ and $P, Q \geq 0$ then $P \geq Q^2$

and if $P \geq Q^2$ and $P, Q \geq 0$ then $\sqrt{P} \geq Q$.

So RTP: $(a^2+c^2)(b^2+d^2) \geq (ab+cd)^2$

Proof: consider the difference

$(a^2+c^2)(b^2+d^2) - (ab+cd)^2$

$= a^2b^2 + a^2d^2 + c^2b^2 + c^2d^2 - (2abcd + a^2b^2 + c^2d^2)$

$= \cancel{a^2b^2} + a^2d^2 + c^2b^2 + \cancel{c^2d^2} - \cancel{a^2b^2} - \cancel{c^2d^2} - 2abcd$

$= a^2d^2 + c^2b^2 - 2abcd$ (2)

$= (ad + cb)^2$

≥ 0 since square

$\therefore (a^2+c^2)(b^2+d^2) \geq (ab+cd)^2$

$\therefore \sqrt{(a^2+c^2)(b^2+d^2)} \geq ab+cd.$ (since $a, b, c, d \geq 0$)

14

b) i) RTP: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let $u = a - x$ then $x = 0 \Rightarrow u = a$

$du = -1dx$ $\quad x = a \Rightarrow u = 0$

$\therefore$ RHS $= \int_0^a f(a-x)\,dx$

$= \int_a^0 f(u) \cdot -1du$ (1)

$= \int_0^a f(u)\,du$

$= \int_0^a f(x)\,dx$ (dummy variable)

b) ii) $\int_0^2 x(2-x)^{10}\,dx$ let $a = 2$

$= \int_0^2 (2-x)(2-(2-x))^{10}\,dx$

$= \int_0^2 (2-x)(x)^{10}\,dx$ (3)

$= \int_0^2 2x^{10} - x^{11}\,dx$

$= \left[\frac{2x^{11}}{11} - \frac{x^{12}}{12}\right]_0^2$

$= \frac{2(2^{11})}{11} - \frac{2^{12}}{12} - (0-0)$

$= \frac{12(2^{12}) - 11(2^{12})}{132}$

$= \frac{2^{12}}{132}$

$= \frac{2^{10}}{33}$

14 cont'd

c) Proof by contrapositive:

RTP: If m, n are integers with a common factor then $m+n$ and $m-n$ also have a common factor.

Proof: Let $m = kp$ and $n = kq$ where p, q are integers and k is an integer and the common factor of m and n,

then $m+n = kp + kq$
$= k(p+q)$ 3

and $m-n = kp - kq$
$= k(p-q)$

$\therefore$ $m+n$ and $m-n$ both have common factor k.

$\therefore$ The contrapositive is true.

$\therefore$ Original statement: if $m+n$ and $m-n$ have no common factors then m and n have none. QED!

d) i)

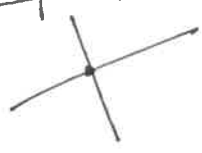
n	2	3	4	5	6
N	1	3	6	10	15

2 3 4 5

2

ii) RTP: $N = \dfrac{n(n-1)}{2}$, $n \geq 2$

Proof: Prove $P(2)$ true.



2 lines, one point of intersection.

$\therefore$ $N = \dfrac{2(2-1)}{2}$
$= \dfrac{2 \times 1}{2}$
$= 1$

$\therefore P(2)$ true.

Assume $P(k)$ is true for some $k \in \mathbb{N}$, ie. $N = \dfrac{k(k-1)}{2}$ ie. if there are k lines then $N = \dfrac{k(k-1)}{2}$ points of intersection

RTP: $P(k+1)$ is true ie. if $k+1$ lines then $N = \dfrac{(k+1)(k+1-1)}{2}$
$= \dfrac{(k+1)k}{2}$ points of intersection.

Proof: Consider adding another line to k lines. This creates another k points of intersection.

$\therefore$ $N = \dfrac{k(k-1)}{2} + k$

$= \dfrac{k(k-1)}{2} + \dfrac{2k}{2}$ 3

$= \dfrac{k^2 - k + 2k}{2}$

$= \dfrac{k^2 + k}{2}$

$= \dfrac{k(k+1)}{2}$

$=$ RHS of $P(k+1)$.

$\therefore P(k+1)$ is true.

$\therefore P(n)$ true $\forall n \in \mathbb{N}, n \geq 2$ by Math Induction.

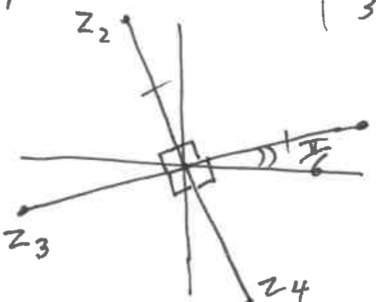
Solutions to
Title: Ascham 2021 Math Ext 2 TRIAL

(7)

Q15

a) Let $z^4 = -\frac{1}{2} + \frac{i\sqrt{3}}{2} = 1 \operatorname{cis} \frac{2\pi}{3}$

$z_1 = $ 1st root $= \operatorname{cis}\left(\frac{2\pi}{3} \div 4\right) = \operatorname{cis} \frac{\pi}{6}$



(4)

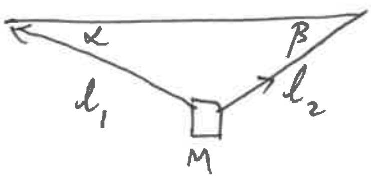
others equally spaced $\frac{2\pi}{4} = \frac{\pi}{2}$ from z_1.

$z_2 = \operatorname{cis}\left(\frac{\pi}{6} + \frac{\pi}{2}\right) = \operatorname{cis} \frac{2\pi}{3}$

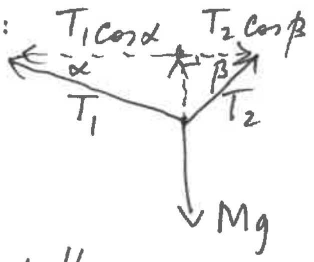
$z_3 = \operatorname{cis}\left(\frac{\pi}{6} + \pi\right) = \operatorname{cis} \frac{7\pi}{6}$

$z_4 = \operatorname{cis}\left(\frac{\pi}{6} - \frac{\pi}{2}\right) = \operatorname{cis}\left(-\frac{\pi}{3}\right).$

b)



Forces:



(4)

Horizontally:

$T_1 \cos\alpha - T_2 \cos\beta = 0$

$\therefore T_1 \cos\alpha = T_2 \cos\beta \Rightarrow T_2 = \frac{T_1 \cos\alpha}{\cos\beta}$

Vertically:

$T_1 \sin\alpha + T_2 \sin\beta - Mg = 0$

$\therefore T_1 \sin\alpha + \frac{T_1 \cos\alpha}{\cos\beta} \cdot \sin\beta = Mg$

$T_1 \sin\alpha \cos\beta + T_1 \cos\alpha \sin\beta = Mg\cos\beta$

$T_1 (\sin\alpha\cos\beta + \cos\alpha \sin\beta) = Mg\cos\beta$

15 b) cont'd:

$T_1 (\sin(\alpha+\beta)) = Mg\cos\beta$

$\therefore T_1 = \frac{Mg\cos\beta}{\sin(\alpha+\beta)}$ QED!

c) $I_n = \int \frac{dx}{(x^2+1)^n}$.

i) RTP: $\int \frac{dx}{(x^2+1)^n} = \int \frac{dx}{(x^2+1)^{n-1}} - \int \frac{x^2\,dx}{(x^2+1)^n}$

Proof: RHS $= \int \frac{dx}{(x^2+1)^{n-1}} - \int \frac{x^2\,dx}{(x^2+1)^n}$

$= \int \frac{(x^2+1)\,dx}{(x^2+1)^n} - \int \frac{x^2\,dx}{(x^2+1)^n}$ (1)

$= \int \frac{\cancel{x^2}+1-\cancel{x^2}\,dx}{(x^2+1)^n}$

$= \int \frac{dx}{(x^2+1)^n} = $ LHS QED!

ii) $I_n = \frac{Ax}{(x^2+1)^{n-1}} + BI_{n-1}$

ie. $I_n = \frac{Ax}{(x^2+1)^{n-1}} + B\int \frac{dx}{(x^2+1)^{n-1}}$.

Now: $I_n = \int \frac{dx}{(x^2+1)^{n-1}} - \int \frac{x^2\,dx}{(x^2+1)^n}$

$= \int \frac{dx}{(x^2+1)^{n-1}} - \int \frac{x \cdot x\,dx}{(x^2+1)^n}$ See over

$= \int \frac{dx}{(x^2+1)^{n-1}} - \left[uv - \int v\,du\right]$ (4)

where $u = x$

$du = 1dx$

$dv = \frac{1}{2} \cdot \frac{2x\,dx}{(x^2+1)^n}$

$= \frac{1}{2} \cdot 2x(x^2+1)^{-n} dx$

P.T.O.

Q15 Cont'd

c) ii) $\displaystyle\int \frac{dx}{(x^2+1)^n} = \int \frac{1\,dx}{(x^2+1)^{n-1}} - \int \frac{x.x\,dx}{(x^2+1)^n}$

Now $\displaystyle\int \frac{1\,dx}{(x^2+1)^{n-1}} = I_{n-1}$

So $\displaystyle\int \frac{x.\,x\,dx}{(x^2+1)^n} = \int u\,dv$

$= uv - \int v\,du$ Let $u = x$

$du = dx$

$dv = x(x^2+1)^{-n}\,dx$

$v = \frac{1}{2(-n+1)}(x^2+1)^{-n+1}$

$= \frac{1}{2(1-n)(x^2+1)^{n-1}}$

$\displaystyle = x.\frac{1}{2(1-n)(x^2+1)^{n-1}} - \int \frac{1\quad dx}{2(1-n)(x^2+1)^{n-1}}$

$\displaystyle = \frac{x}{2(1-n)(x^2+1)^{n-1}} + \frac{1}{2n-2}\int \frac{1\,dx}{(x^2+1)^{n-1}}$

$\therefore I_n = I_{n-1} - \frac{x}{2(1-n)(x^2+1)^{n-1}} - \frac{1}{2n-2}I_{n-1}$

$= \frac{x}{(2n-2)(x^2+1)^{n-1}}$

$+ \frac{(2n-2)I_{n-1} - 1\,I_{n-1}}{2n-2}$

$= \frac{x}{(2n-2)(x^2+1)^{n-1}} + \frac{2n-3}{2n-2}I_{n-1}$

$\therefore A = \frac{1}{2n-2}$, $B = \frac{2n-3}{2n-2}$

Q15 cont'd

c) iii) $\lim_{k\to\infty}\int_0^k \frac{dx}{(x^2+1)^2}$ $\quad n=2$

$$= \lim_{k\to\infty}\left(\frac{1}{2(2)-2}\left[\frac{x}{(x^2+1)^1}\right]_0^k + \frac{2(2)-3}{2(2)-2}\int_0^k \frac{1}{(x^2+1)^1}\,dx\right]$$

$$= \lim_{k\to\infty}\left[\frac{1}{2}\left[\frac{k}{k^2+1}-0\right] + \frac{1}{2}\left[\tan^{-1}x\right]_0^k\right]$$

$$= \frac{1}{2}(0) + \frac{1}{2}\left(\frac{\pi}{2}-0\right)$$

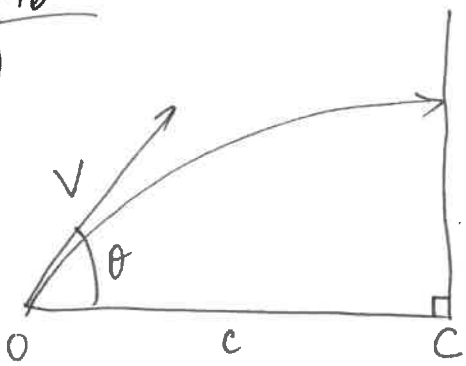
$$= \frac{\pi}{4}$$

(2)

Solutions to
Title: Ascham 2021 Math Ext 2 Trial

Q16

a)



$t=0$
$x=0$
$y=0$
$\dot{x}=V\cos\theta$
$\dot{y}=V\sin\theta$

Let $\ddot{x}=0$, $\ddot{y}=-g$

$\dot{x}=\int 0\,dt$, $\dot{y}=\int -g\,dt$
$= C_1$ $\quad = -gt + C_2$

When $t=0$,
$\dot{x}=V\cos\theta = C_1$, $\dot{y}=V\sin\theta = -g(0)+C_2$

$\therefore \dot{x}=V\cos\theta$, $\dot{y}=-gt+V\sin\theta$

$x=\int V\cos\theta\,dt$, $y=\int -gt+V\sin\theta\,dt$

$x = Vt\cos\theta + C_3$, $y=\frac{-gt^2}{2}+Vt\sin\theta+C_4$

When $t=0$,
$x=0=V(0)\cos\theta + C_3 \Rightarrow C_3=0$,
$0=y=\frac{-g(0)^2}{2}+V(0)\sin\theta+C_4$
$\Rightarrow C_4=0$

$\therefore x=Vt\cos\theta$, $y=\frac{-gt^2}{2}+Vt\sin\theta$.

Now want horizontal range to be at least c metres.
Range Max when $y=0$.

$\therefore 0=\frac{-gt^2}{2}+Vt\sin\theta$

$0=t\left(\frac{-gt}{2}+V\sin\theta\right)$

$\therefore t=0$ or $\frac{gt}{2}=V\sin\theta$

$\therefore t=\frac{2V\sin\theta}{g}$

Q16
a) cont'd

$\therefore x = Vt\cos\theta$
$= V\left(\frac{2V\sin\theta}{g}\right)\cos\theta$
$= \frac{V^2\sin 2\theta}{g}$

Now $\frac{V^2\sin2\theta}{g} > c$

Since Max $\sin2\theta = 1$ then

$\frac{V^2}{g} > c$ (4)

$\therefore V^2 > cg$

Since $V, c, g > 0$ then

$V > \sqrt{cg}$. QED!

b) i) RTP:
$1+r+r^2+\ldots+r^{n-1} = \frac{1-r^n}{1-r}$, $r\neq 1$

$1+r+r^2+\ldots+r^{n-1} = \frac{a(1-r^n)}{1-r}$

Since G.P. now $a=1$ so

$1+r+r^2+\ldots+r^{n-1} = \frac{1(1-r^n)}{1-r}$

$= \frac{1-r^n}{1-r}$. QED! $(r\neq 1)$

[G.P. since $\frac{T_3}{T_2}=\frac{r^2}{r}=r$, $\frac{T_2}{T_1}=\frac{r}{1}=r$] (1)

ii) $\lim_{n\to\infty} 1+r+r^2+\ldots+r^{n-1} = \lim_{n\to\infty}\frac{1-r^n}{1-r}$

$= \frac{1-0}{1-r}$ for $0<r<1$ (1)

$= \frac{1}{1-r}$

16 b) cont'd

iii) For $0 < x < 1$

RTP: $x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots + \frac{x^n}{n} + \ldots = -\ln(1-x)$.

Proof: From (ii)

$1 + r + r^2 + \ldots + \ldots = \frac{1}{1-r}$ for $0 < r < 1$

$\therefore \int_0^x 1 + r + r^2 + \ldots + \ldots \, dr = \int_0^x \frac{1}{1-r} \, dr$

$\therefore \left[r + \frac{r^2}{2} + \frac{r^3}{3} + \ldots + \frac{r^n}{n} + \ldots \right]_0^x = -\left[\ln|1-r| \right]_0^x$

$\therefore x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots + \frac{x^n}{n} + \ldots - (0 + 0 + 0 + \ldots)$

$= -\left[\ln|1-x| - \ln|1-0| \right]$

$\therefore x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots + \frac{x^n}{n} + \ldots = -\ln|1-x| + 0$

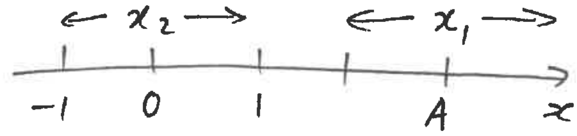
$= -\ln|1-x|$.

QED !!!

2

Q16 contd

c)



$x_1 = A + \sin nt$

$\ddot{x}_2 = -n^2 x_2.$

$t=0, \quad x_1 = A, \quad x_2 = 1.$

i) RTP: $\dot{x}_1^{\,2} = n^2\left(1-(x_1-A)^2\right)$

Proof: $x_1 = A + \sin nt$

$\dot{x}_1 = n\cos nt$

$\therefore \dot{x}_1^{\,2} = n^2\cos^2 nt$

(2)

$x_1 - A = \sin nt$

$\therefore (x_1-A)^2 = \sin^2 nt$

Now $\cos^2 nt + \sin^2 nt = 1$

$\therefore \dot{x}_1^{\,2} = n^2\cos^2 nt$

$\therefore \dot{x}_1^{\,2} = n^2(1-\sin^2 nt)$

$= n^2\left(1-(x_1-A)^2\right)$ QED!!!

ii) RTP: $\dot{x}_2^{\,2} = n^2(1-x_2^2)$.

Proof: $\ddot{x}_2 = -n^2 x_2$

$\therefore \frac{d}{dx}\left(\frac{1}{2}v_2^2\right) = -n^2 x_2$ (2)

$\frac{1}{2}v_2^2 = \int -n^2 x_2 \, dx_2$

$\frac{1}{2}v_2^2 = \frac{-n^2 x_2^2}{2} + C$

When $t=0$, $x_2 = 1$, $v_2 = 0$:

$0 = \frac{-n^2(1)^2}{2} + C$

$\therefore C = \frac{n^2}{2}$

Q16 c) ii) contd:

$\therefore \frac{1}{2}v_2^2 = \frac{-n^2 x_2^2}{2} + \frac{n^2}{2}$

$\therefore v_2^2 = -n^2 x_2^2 + n^2$

$\therefore v_2^2 = n^2(1-x_2^2)$

$\therefore \dot{x}_2^{\,2} = n^2(1-x_2^2)$ QED!!

iii) Since $\ddot{x}_2 = -n^2 x_2$ we know the centre of motion is 0. Also, when $t=0$, $v_2 = 0$ when $x_2 = 1$ $\therefore$ P_2 is oscillating around 0 with amplitude 1 starting at 1 $\therefore$ convenient equation is $x_2 = 1\cos nt$.

Since x_1 is oscillating around A where $A > 0$ then minimum distance is given by $x_1 - x_2$ minimised.

$\therefore x_1 - x_2 = A + \sin nt - \cos nt$

$\frac{d}{dt}(A + \sin nt - \cos nt) = 0$ (3)

$n\cos nt + n\sin nt = 0$

$\sin nt = -\cos nt$

$\tan nt = -1 \qquad t \geq 0$

$nt = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}, \ldots$

$t = \frac{3\pi}{4n}, \frac{7\pi}{4n}, \frac{11\pi}{4n}, \frac{15\pi}{4n}, \ldots$

We can see from sketch that minimum is when $t = \frac{7\pi}{4n}$.

PTO

Q16 cont'd

c) iii) cont'd

$\therefore$ Min distance

$= A + \sin\cancel{x}\left(\frac{7\pi}{4\cancel{x}}\right) - \cos\cancel{x}\left(\frac{7\pi}{4\cancel{x}}\right)$

$= A + -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= A - \frac{2}{\sqrt{2}}$

$= A - \sqrt{2}$ assuming $A > \sqrt{2}$!